## Fractional charge in transport through a 1D correlated insulator of finite length.

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Transport through a one channel wire of length L confined between two leads is examined when the 1D electron system has an energy gap 2M:  $M > T_L \equiv v_c/L$  induced by the interaction in charge mode ( $v_c$ : charge velocity in the wire). In spinless case the transformation of the leads electrons into the charge density wave solitons of fractional charge q entails a non-trivial low energy crossover from the Fermi liquid behavior below the crossover energy  $T_x \propto \sqrt{T_L M} e^{-M/[T_L(1-q^2)]}$  to the insulator one with the fractional charge in current vs. voltage, conductance vs. temperature, and in shot noise. Similar behavior is predicted for the Mott insulator of filling factor  $\nu = integer/(2m')$ .

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Fractional charge (FC) is one of the central issues in the physics of strongly correlated electronic system. Recent measurements of the shot noise [1] have shown the existence of the quasi-particles with charge  $q = \pm e/3$  in fractional quantum Hall liquid (FQHL) at filling  $\nu = 1/3$ . In this incompressible system the quantum transport is dominated by the edge mode which has been described as a chiral Tomonaga-Luttinger liquid (TLL). Another realization of TLL is the quantum wire made out of GaAs. Recently Tarucha et al. succeeded to introduce a periodic potential to it, which allows Umklapp scattering and, hence, the insulating behavior due to correlation. For the spinless electrons such a 1D insulator is also a charge density wave insulator (CDW). Transport through it reduces to mutual transformations between the reservoir electrons and the CDW (anti)solitons of FC  $q < e \equiv 1$  and mass M. The uniform model of the 1D CDW insulator [5,6] has been developed earlier and addressed to the macroscopic quasi 1D organics and  $NbSe_3$ where the quantum tunneling motion has been observed. To observe FC, the contacts to leads play essential role. Chamon and Fradkin [7] argued that the equilibration along the wide contacts is necessary to have the fractional quantized conductance in FQHL. It does not occur in the quantum wire through point-like contacts. Then the fractional charge does not appear in conductance [8] or shot noise [9] in the low energy limit. However, the absence of the equilibration gives rise to rich crossovers, where FC manifests itself in many ways.

In this paper we consider transport through a finite length L wire connected to the source and drain Fermi liquid reservoirs, when the 1D electrons of the wire are in the Mott insulating phase characterized by the gap  $2M \gg v_c/L \equiv T_L$  in the energy spectrum ( $v_c$ : charge velocity in the wire). In spinless case we describe the transport with an inhomogeneous sin-Gordon action after implementing bosonization and specification of the parameters [10] from comparison with the Bethe-anzats solution [11] known for the uniform sin-Gordon model. For low energy (< M), solution of the inhomogeneous

model may be done with the quasiclassics [12] transformed into the instanton technique [13]. Up to a quantum renormalization of the soliton mass M this procedure is known to be exact. The (anti)solitons in this model see the electron reservoirs as a slowly decaying interaction along the boundaries whose strength is proportional to  $q^2$ , meanwhile the electrons see the CDW condensate as quantization of their phase at the boundaries whose values  $2\pi q \times integer$  relate to the CDW vacua. At energies less than  $T_L$  the sweeps of the solitons switching the CDW phase are deemed instant. Then the problem reduces through the Duality Transform [14] to the point scatterer in the TLL whose solution has been known after [15,16]. Behavior of the low temperature conductance is ruled by the scaling dimension of the tunneling operator. The latter is always relevant for the correlated insulator opposed to the band insulator where it is marginal. Hence the correlated insulator conductance recovers its free electron value in the absence of impurities passing through a crossover at the temperature  $T_x = cst\sqrt{T_L M} exp[-\varepsilon/(1-g)], \ \varepsilon =$  $\sqrt{M^2-(q\mu)^2}/T_L,\ g=q^2$ , ( $\mu$ : chemical potential). Similar, the current increases linearly with increase of the voltage below  $T_x$  and then goes down as a negative power of the voltage. The tunneling scaling dimension q is related to the quantization of the condensate charge which is always fractional except for the case of the band insulator. This FC turns up in the shot noise of the current at energies above  $T_x$ . Below  $T_x$ , however, the Fermi liquid behavior occurs [9,17]. This description of the CDW condensate is also applicable to the charge mode of electrons with spin at rational fillings of even denominator  $\nu = 1/(2m') \times integer$ . Therefore, above results may be addressed to transport through the Mott insulator after redefinitions: q = 1/m',  $g = q^2/2$ . In particular, for the Mott-Hubbard insulator  $\nu = 1/2$  the charge q becomes integer. This marginality shows up in saturation of the current as voltage exceeds  $T_x$  in agreement with [18].

A general Hamiltonian for spinless electrons moving in the periodic potential  $V_{period}(x)$  (period a) of a one

channel wire 0 < x < L may be written after [8]

$$\mathcal{H} = \int_{-\infty}^{\infty} dx \{ \psi^+(x) \left( -\frac{\partial_x^2}{2m^*} - E_F \right) \psi(x) + u_0 \varphi(x) \rho^2(x) + V_{period}(x) \rho(x) \}$$
(1)

where the wire is assumed to be smoothly connected to the right and left reservoirs. The Fermi momentum  $k_F$  and the Fermi energy  $E_F$  is determined by the filling factor  $\nu$  as  $\nu = k_F a/\pi$  and  $E_F \approx v_F k_F$ . In Eq. (1) the function  $\varphi(x) = \theta(x)\theta(L-x)$  switches on the electron-electron interaction inside the wire. Follow-

ing Haldane's generalized bosonization procedure [19] to account for the nonlinear dispersion one has to write the fermionic fields as  $\psi(x) = \sqrt{k_F/(2\pi)} \sum exp\{i(n+1)(k_Fx+\phi(x)/2)+i\theta(x)/2\}$  and the electron density as  $\rho(x) = (\partial_x \phi(x) + 2k_F)/(2\pi) \sum exp\{in(k_Fx+\phi(x)/2)\}$  where summation runs over even n and  $\phi$ ,  $\theta$  are mutually conjugated bosonic fields  $[\phi(x), \theta(y)] = i2\pi sgn(x-y)$ .

After substitution of these expressions into (1) the Hamiltonian takes its bose-form. The associated Eucledian action with this Hamiltonian can be constructed considering the spatial derivative of the  $\theta$  field as a momentum to  $\phi$ . Its Lagrangian density reads:

$$\mathcal{L} = \frac{v(x)}{2g(x)} \left\{ \frac{1}{v^2} \left( \frac{\partial_{\tau} \phi(\tau, x)}{\sqrt{4\pi}} \right)^2 + \left( \frac{\partial_x \phi(\tau, x)}{\sqrt{4\pi}} \right)^2 \right\} - \frac{E_F^2 U_m}{2\pi v_F} \varphi(x) \cos(m[\phi(\tau, x) + 2k_{mF} x])$$
 (2)

where we assumed that deviation of the chemical potential from that one  $(\mu = 0)$  corresponding to  $\nu =$  $1/m \times integer$  is small and only one Umklapp process involving m electron scattering of  $2mk_{mF}$  momentum transfer is relevant. Here  $v_c$  is the charge velocity different from the Fermi one because of the interaction inside the wire: for  $x \in [0, L]$  the strength of the forward scattering g(x) = g, 0 < g < 1 and  $v_c = v_F/g$ . Outside the wire g(x) = 1 and  $v(x) = v_F$ . The strengths of the forward g and Umklapp  $U_m$  scattering inside the wire may be related to the initial parameter  $u_0$  in Eq. (1). When the Umklapp interaction opens up the gap in the spectrum the parameter  $k_{mF}$  does not mean the momentum transfer as in the perturbative case. In general, it specifies the value of the chemical potential as  $\mu = k_{mF}v_c/g$ .

In the uniform case of the infinite wire above action coincides with the sin-Gordon one, which properties are known from the Renormalization Group (RG) procedure [11] extended with the Bethe ansatz solution beyond the perturbation theory. The velocity v fixes symmetry between x and  $v\tau$  and does not renormalize with decreasing the energy scale. There are two standard characteristics:  $K = gm^2 - 2$ ,  $W = |U_m v_c/v_F|$  of the RG-flow at zero  $k_{mF}$ . A hidden symmetry fixes two separatrices  $W = \pm K$ . The line K = -1 relates to fermionization [20]. Then the intersection W = -K = 1 gives the point which attracts all trajectories of the massive phase. These trajectories cover part of the upper half-plane of the parameters where K > -2 except for K > W where W renormalizes into zero and the TLL phase occurs. Introduction of a non-zero  $k_{mF}$  is not important for the TLL phase. In the massive phase the chemical potential for the sin-Gordon elementary excitations, solitons destroys the gap on exceeding the soliton mass. Comparison of the parameters of (2) with the Bethe-anzats solution shows that the local interaction producing a gap has to be strong enough for the spinless electron system. In the quantum wire such a local interaction occurs if the

wire width w is about the distance d from the wire to the screening gate. In the opposite case of the long range interaction the initial value of the constant  $g \propto 1/\sqrt{d/w}$ is small [21] and the uniform sin-Gordon model (2) with m larger than 2 may acquire the gap at appropriate rational filling. The same sin-Gordon action (2) describes also the even Umklapp processes in the 1D system of the interacting electrons with spin [2], if the  $\phi$  field denotes the charge bose field, m under the  $\cos$  in (2) is changed into  $m/\sqrt{2}$  and  $k_{mF}$  into  $\sqrt{2}k_{mF}$ . Contrary to the spinless case, any repulsive interaction will open a gap in the spectrum of the 1D spin electron system at half-filling m=2 since q=1 is critical: K=0. Carrying out all consideration below for the spinless electrons, we will notice a necessary modification in the spin case at the end.

Duality Transform - In the massive phase of the wire where  $\mu$  is much less than the mass M of the solitons the quasiclassical method gives correct physical picture [12] up to a quantum renormalization of the parameters. Then tunneling of the charge through the wire may be described in the instanton technique [13]. First, let us integrate out the field  $\phi$  in the reservoirs  $x \in [0, L]$  in (2). It can be done, say, for  $x \leq 0$  with introducing a new variable:  $\xi(\tau, x) = \phi(\tau, x) - \phi(\tau, 0)e^{x/L'}$  satisfying the boundary condition  $\xi(\tau,0)=0$ . Here parameter  $L'\to\infty$ regularizes the infinite length of the left reservoir. Substitution of the new variable into (2) brings up a Gaussian action for  $\xi$ , which after integration produces the boundary action describing non-local interaction for  $\phi(\tau,0)$ . Repeating the same procedure for  $x \geq L$  we can write this boundary action as

$$S_{int} = \frac{Re}{16\pi^2} \sum_{y=0} \int \int d\tau d\tau' \left( \frac{\phi(\tau, y) - \phi(\tau', y)}{\tau - \tau' - i\alpha} \right)^2$$
(3)

To simplify notations we will drop out  $v_c$  below. Then all energies become transformed into reverse lengths in line with:  $length^{-1} = energy/v_c$ . It assumes, in particular, rescaling  $\tau$  in (3) so that a new one is previous  $v_c\tau$ , and

$$S_{blk} = \int d\tau \{ \frac{1}{4\pi g m^2} \int_0^L dx \frac{m^2}{2} [(\partial_\tau \phi(\tau, x))^2 + (\partial_x \phi(\tau, x))^2] + \lambda^2 (1 - \cos(m\phi(\tau, x))) - \frac{k_{mF}}{2g\pi} [\phi(\tau, L) - \phi(\tau, 0)] \}$$
(4)

In this "bulk" part of the action where  $\lambda^2 = 2gWm^2/\alpha^2$ and the definition of the  $\phi$  field has been changed with respect to (2) to absorb  $2k_{mF}x$  function, the cos term tends to fix this new field  $\phi$  at one of the  $integer \times 2\pi/m$  values. These values correspond to the infinite set of degenerate vacua of the infinite length sin-Gordon model. At finite length, however, the non-degenerate ground state is recovered due to an exponentially weak tunneling between the vacua. The tunneling may be conventionally described in the instanton technique [13]. In this technique the partition function:  $\mathcal{Z} = \int D\phi exp[-S_{blk}]$ that will be considered first for the action (4) omitting the boundary interaction is calculated in an "advanced" saddle point approximation. The extremum function, (anti-)instanton, minimizing the action (4) coincides with the sin-Gordon stationary (anti)soliton and takes the form:  $\pm \phi_0(\tau, x|\tau_0) = m^{-1} f(\lambda[(x-L/2)\cos \varpi (\tau - \tau_0)\sin \varpi$ ]), f(x) = 4arctan(exp(x)). The angle  $\varpi \in [0,\pi)$  satisfies:

$$S_{blk}\{\pm\phi_0\} = \frac{L}{\sin\varpi} \left( M \pm \frac{k_{mF}\cos\varpi}{gm} \right) \equiv Ls_0$$
 (5)

$$\partial_{\varpi} s_0 = 0$$
: 
$$\begin{cases} \cos \varpi = \mp k_{mF}/(gmM) = \mp q\mu/M \\ s_0 = M \sin \varpi = \sqrt{M^2 - (q\mu)^2} \end{cases}$$
 (6)

being different for the instanton and anti-instanton. Here the mean field value of M is a classical (anti)soliton mass  $M_{cl} = 2\lambda/(\pi m^2 g)$  and the minimum of the action  $S_{blk}$ exists while  $M > k_{mF}/(gm) = q\mu$ . The instanton/ antiinstanton varies the phase  $\phi$  by  $\mp 2\pi/m$  and carries the charge  $\pm q$ , q = 1/m. Both limit values of  $\phi$  are approached exponentially as  $\tau$  moving away from the kink location  $\tau_0$ . Therefore, a many (anti-)instanton function compiled from a number of kinks which locations in imaginary time  $\tau_i$  are well separated from each other, presents an approximate minimum function for the action (4). The partition function reduces to a sum of the contributions of all these minima. The contribution to the functional integral of  $\mathcal{Z}$  may be found expanding the action around each (anti-)instanton ( $e_i = \pm$ ) of the many kink function to the second order and considering each kink independently

$$\mathcal{Z} = cst(1 + \sum_{N=1}^{\infty} 1/N! \int \prod_{j=1}^{N} \{ d\tau_j \sum_{e_j = \pm} p_{e_j} e^{-Ls_0} \}) \quad (7)$$

Here the cst has absorbed the infinite sum over all degenerate vacua. The prefactors  $p_{\pm}$  for the (anti-)instanton are equal and may be calculated as

$$p_{\pm} = \sqrt{\int dx \int d\tau \frac{[\phi_0'(\tau, x)]^2}{4\pi g m^2}} \left\{ \frac{Det(-\partial^2 + \lambda^2)}{Det'(-\partial^2 + \lambda^2 \cos \phi_0)} \right\}^{1/2}$$

$$= \sqrt{\frac{M\sin\varpi}{m^2L}} e^{-L\Delta M/\sin\varpi}$$
 (8)

Here the determinant Det of the first operator  $-\partial^2 + \lambda^2 \equiv$  $-\partial_{\tau}^{2}-\partial_{x}^{2}+\lambda^{2}$  has to be calculated in the strip. The same has to be done for determinant Det'. However, the latter does not include contribution of the zero energy mode of the second Shrödinger operator  $-\partial^2 + \lambda^2 \cos \phi_0$ . This operator differs with the first one due to an additional potential well produced by the instanton. Equality of the lowest eigenvalue of the second operator to zero ensues from the translational invariance of the action (4) in time. The result of calculation of (8) p has an exponent with the one loop quantum correction to the classical soliton mass [12]:  $\Delta M = \sum \Delta \omega / 2$  multiplied by L where  $\Delta \omega$  is difference between the eigenvalues of the operators depending on  $\tau$  only and constructed from the first and second one, respectively, by dropping out their x dependence. This correction renormalizes the value of the mass  $M = M_{cl}$ in (5) found earlier and, hence, the value of the angle  $\varpi$ in (6). One can see that taking into account the higher order corrections amounts to the complete RG-procedure for the uniform sin-Gordon action which ends up with Mequal to the quantum soliton mass and the renormalized value of  $q: q \to 1/m^2$  in above equations.

Incorporation of the boundary interaction (3) in above consideration entails following changes in the instanton representation of the partition function (7). Besides a dimensionless factor multiplying the ratio of the determinants due to their boundary variation, there appears an additional contribution to the one kink mean field action together with the direct interaction between the kinks. The infrared divergency in these terms limits the sum over  $e_j = \pm$  to the neutral combinations:  $\sum_j e_j = 0$ , for which the boundary part of the exponent becomes  $S_{int} = \frac{-1}{m^2} \left[ \sum_{i \neq j}^N e_i e_j F_{e_i e_j} (\tau_i - \tau_j) - N ln(M \sin \varpi) \right]$ . Both interactions are equal  $F_{\pm}(\tau) = ln\sqrt{(\tau_i - \tau_j)^2 + 1/D'^2}$  at long time  $\tau > 1/D'$ , but differ at short. At  $\sin \varpi = 1$ the time 1/D' comes about through bending the straight linear form of the kinks on their approaching each other. It can be evaluated as  $D' = \sqrt{T_L M}$ . For  $q\mu \gg \sqrt{T_L M}$ the time is approximately an extension of the single kink  $L/\tan \omega$  and  $D' = T_L \sqrt{(M/q\mu)^2 - 1} \approx T_L M/(q\mu)$ . Integrating out the energies > D' in (7) results in renormalization of the prefactor P(D'). In the lowest order in exponentially small kink amplitude only the kink-antikink pairs contribute. The renormalized prefactor is

$$P = p \times e^{-\frac{1}{m^2} \left[ 2 \int_{D'}^{M \sin \varpi} F_{-}(\omega) d\omega - F_{-}(\tau=0) - \ln(M \sin \varpi) \right]}$$

If at  $\sin \varpi = 1$  the function  $F_-(\tau)$  quickly drops reaching the bottom value  $F_-(\tau) = -lnM$  at  $\tau < 1/D'$ , this equation yields  $P \simeq p = cstD'$ , which complies with the exact solution  $P = D' \sin \varpi/\pi$  known for the band insulator m = 1. On the other hand, assuming correctness of the Gaussian approximation, the renormalized prefactor may be found in general without specification of F in above scheme as  $P = cst \times D' \sin \varpi (\sqrt{T_L M/ \sin \varpi}/D')^{1-1/m^2}$ .

Low energy transport - Constructed Dual Transform of the low energy partition function may be identified with the partition function for the point scatterer in the TLL [15] in the following way. We introduce a bosonic field  $\theta(\tau,x)$  after Schmid [14] and will ascribe to each (anti-)instanton in (7) a factor  $exp\{\mp\theta(\tau_j,0)/m\}$ . Having a small voltage V been applied, the real time Lagrangian for the  $\theta$  field reads

$$\int dx \mathcal{L}_t = \int \frac{dx}{2} \left\{ \left( \frac{\partial_t \theta(t, x)}{\sqrt{4\pi}} \right)^2 - \left( \frac{\partial_x \theta(t, x)}{\sqrt{4\pi}} \right)^2 \right\}$$
$$- \frac{D'A}{\pi} \cos((Vt/v_c + \theta(t, 0))/m) \tag{9}$$

where t has a length dimension and the amplitude of the fractional charge tunneling  $A=2\pi Pe^{-LM\sin\varpi}/D'$  is exponentially small. The tunneling current comes about through varying the tunneling part of the Hamiltonian associated to (9):  $J=\frac{\delta}{\delta\theta}\mathcal{H}_{tunn}=-AD'\sin((\theta+Vt)/m)/(\pi m)$ . On rescaling  $\tilde{\theta}=\theta/m$ , the tunneling may be regarded as a point scatterer of the integer charge in the uniform TLL with  $g=m^{-2}=q^2$ . Solution of this problem first considered asymptotically [15] has been later thoroughly examined in the Bethe anzats technique [16]. In this problem the current J can be identified as the m times backscattering current under applied mV voltage: both the backscattering current and the effective voltage are proportional to g [9] in the uniform TLL. Its zero temperature expression takes a closed form:

$$\frac{J}{V\sigma_0} = \frac{i}{\sqrt{\pi}} \int_C \frac{dz}{z} \frac{\Gamma(1 + \frac{z}{1-g})\Gamma(1 + \frac{z}{1-1/g})}{4\Gamma(3/2 + z)} \left(\frac{mV}{T_x}\right)^{2z}$$
(10)

where the contour C goes along the imaginary axis circumventing the zero from the left,  $\sigma_0$  is equal to  $1/(2\pi)$  in spinless case and  $T_x = \frac{2}{g}D'(A/\Gamma(g))^{1/(1-g)}$  [22] gives a crossover energy. Substituting the value of A, one can find  $T_x \simeq cst\sqrt{T_LM}e^{-M\sin\varpi/[(1-g)T_L]}$ . Below  $T_x$  the conductance J/V approaches its universal value  $\sigma_0$  as  $\sigma_0(1-c_1(1/g)\times (mV/T_x)^{2(1/g-1)})$  and  $c_1(g)=\sqrt{\pi}\Gamma(g+1)/(2\Gamma(1/2+g))$ . Above  $T_x$  it is going to zero as  $\sigma_0c_1(g)(mV/T_x)^{2(g-1)}$  reaching an exponentially small value at  $V\approx T_L$ . The linear bias conductance vs. temperature relates to above voltage dependence through exchange  $T\leftrightarrow V$  with the same exponents in the low and high temperature asymptotics. In the band insulator case m=1 the crossover temperature  $T_x$  is zero and the conductance reduces to the constant:  $\sigma_0A^2$ . For

larger values of m, however, it becomes non-monotonic function of V with a maximum at  $V \approx T_x$ .

Next, let us examine the shot noise of the current  $\delta J^2$ . Since at T=0 it coincides with the zero frequency noise of the backscattering current [9], we may read off the result from the shot noise of the point scatterer in the TLL [23,22] as:  $\delta J^2(V) = -\frac{mV^2}{2(1/g-1)} \partial_V \frac{J}{V}$ . In the high voltage  $mV > T_x$  regime this equation reveals that the charge of the transport carrier is fractional q:  $\delta J^2 = qJ$ , while for the low voltage the equation takes its Fermi liquid form:  $\delta J^2 = m(\sigma_0 V - J)$ . The factor m in the latter case shows that the backscattering is the *m*-electron process. Addition of a weak impurity scattering changes the leading low energy asymtotics into the m=1 form. However, the high voltage form remains the same since the instanton representation of tunneling between the CDW vacua holds on in the presence of a weak one electron backscattering after a slight modification of the (anti-)instanton mass and the tunneling amplitude. The last speculation concurs with the earlier considerations [5,6] of the infinite Peierls CDW transport, and can be confirmed with the exact solution for 1D transport through the Mott-Hubbard insulator with impurity [24].

Modification of above consideration for the spin case of filling factor with even denominator  $\nu = 1/m = 1/(2m')$ ensues from above correspondence between spinless and spinful cases:  $m \to m/\sqrt{2}$ ;  $k_{mF} \to \sqrt{2}k_{mF}$  and has to account for an additional factor  $\sqrt{2}$  in relation between the charge density and the fields  $\phi, \theta$  variation. Therefore,  $q=2/m, g=q^2/2$  and  $k_{mF}/m$  transforms into  $q\mu$ as before for the definition of the chemical potential remains the same. Voltage V in (9) acquires the factor  $\sqrt{2}$  which together with another  $\sqrt{2}$  after rescaling m gives correct combination qV under cos. All equations below (9) have no change in their form assuming that above parameters are modified and  $\sigma_0$  becomes  $1/\pi$ . So, there is no change in the shot noise expressions. In spite of the similar form, the results for the Mott insulator have more complicated physics behind, as the reservoir electrons now transform into two types of quasiparticles carrying either spin or charge. In particular, the integer charge (q = 1) of the charge quasiparticle of the Mott-Hubbard insulator m' = 1 does not assume absence of the non-trivial crossover [18] anymore.

Finally, the constructed low energy solution brings up a new insight on the earlier developed theory of transport in the infinite 1D CDW [5,6]. It shows that as the conductance tends to vanish at the energies  $\approx T_L$ , the CDW phase becomes fixed by the reservoirs. The latter has been a crucial assumption for deriving the thermally activated conductivity in the CDW materials. On the other hand, recovering of the conductance below  $T_x$  calls up an earlier Frölich's idea of the CDW sliding in the absence of impurities [25].

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